$$\int_0^{\frac{Log3}{2}} \frac{dt}{1+e^{2t}} = \int_0^{\frac{Log3}{2}} (1-\frac{e^{2t}}{1+e^{2t}})dt = \int_0^{Log\sqrt{3}} (1-\frac{1}{2}\frac{2e^{2t}}{1+e^{2t}})dt$$

$$= \left[t - \frac{1}{2} Log(1+e^{2t}) \right]_0^{\log\sqrt{3}} = Log\sqrt{3} - \frac{1}{2} Log4 + \frac{1}{2} Log2$$

$$= Log\sqrt{3} - \frac{1}{2} Log2 = \frac{1}{2} Log(\frac{3}{2})$$

b) d'après 2°/ on a : $\forall t \in \mathbf{R}$; $(1+e^{2t})f'(t) + (e^{2t}-1)f(t) = -1$

$$\Rightarrow f'(t) = -\frac{1}{1+e^{2t}} - \frac{e^{2t}-1}{1+e^{2t}} f(t) = -\frac{1}{1+e^{2t}} - \frac{(e^{2t}+1)-2}{1+e^{2t}} . f(t)$$

$$= \frac{-1}{1+e^{2t}} - f(t) + \frac{2f(t)}{1+e^{2t}} \; ; J = \int_0^{\frac{Log3}{2}} \frac{1+e^{t}}{(1+e^{2t})^2} dt = \int_0^{\frac{Log3}{2}} \frac{f(t)}{1+e^{2t}} dt$$

or on a montré que $\forall t \in \mathbf{R}$; $f'(t) = -\frac{1}{1+e^{2t}} - f(t) + \frac{2f(t)}{1+e^{2t}}$

d'où $\frac{f(t)}{1+e^{2t}} = \frac{1}{2}\left[f'(t) + \frac{1}{1+e^{2t}} + f(t) \right]$ d'où

$$J = \frac{1}{2} \int_0^{\frac{Log3}{2}} f'(t)dt + \frac{1}{2} \int_0^{\frac{Log3}{2}} \frac{1}{1+e^{2t}} dt + \int_0^{\frac{Log3}{2}} f(t)dt$$

$$= \frac{1}{2}\left[f(t) \right]_0^{Log\sqrt{3}} + \frac{1}{2}(\frac{1}{2} Log\frac{3}{2}) + \frac{1}{2} I$$

$$= \frac{1}{2}(\frac{1+\sqrt{3}}{1+(\sqrt{3})^2} - 1) + \frac{1}{4} Log(\frac{3}{2}) + \frac{\pi}{24} + \frac{1}{4} Log(\frac{3}{2})$$

$$= \frac{\sqrt{3}-3}{8} + \frac{\pi}{24} + \frac{1}{2} Log(\frac{3}{2})$$